Unemployment Fluctuations

Pascal Michaillat
https://pascalmichaillat.org/c1/

Objective :

- countercyclical unemployment
- procyclical tightness
- "large" unemployment fluctuating

→ elasticity of unemployment rate wrt shock is the same as in data

Two wage functions :

- Rigid wage
- Bargained wage

Matching model with rigid wage

$W = \omega \cdot a^{\gamma}$ $\quad\quad \gamma \in [0,1]$

$$L^s(\theta) = \frac{f(\theta)}{s + f(\theta)} \cdot H$$

$$L^d(\theta) = \left[\frac{a \cdot \alpha}{\underline{w \cdot a^{\gamma}} \cdot [1 + \tau(\theta)]^{\alpha}} \right]^{1/1-\alpha}$$

$$L^d(\theta) = \left[\frac{a^{1-\gamma} \cdot \alpha}{w \cdot [1 + \tau(\theta)]^{\alpha}} \right]^{1/\underline{1-\alpha}}$$

$1 - \gamma > 0$

<u>equilibrium condition</u> : $L^s(\theta) = L^d(\theta)$

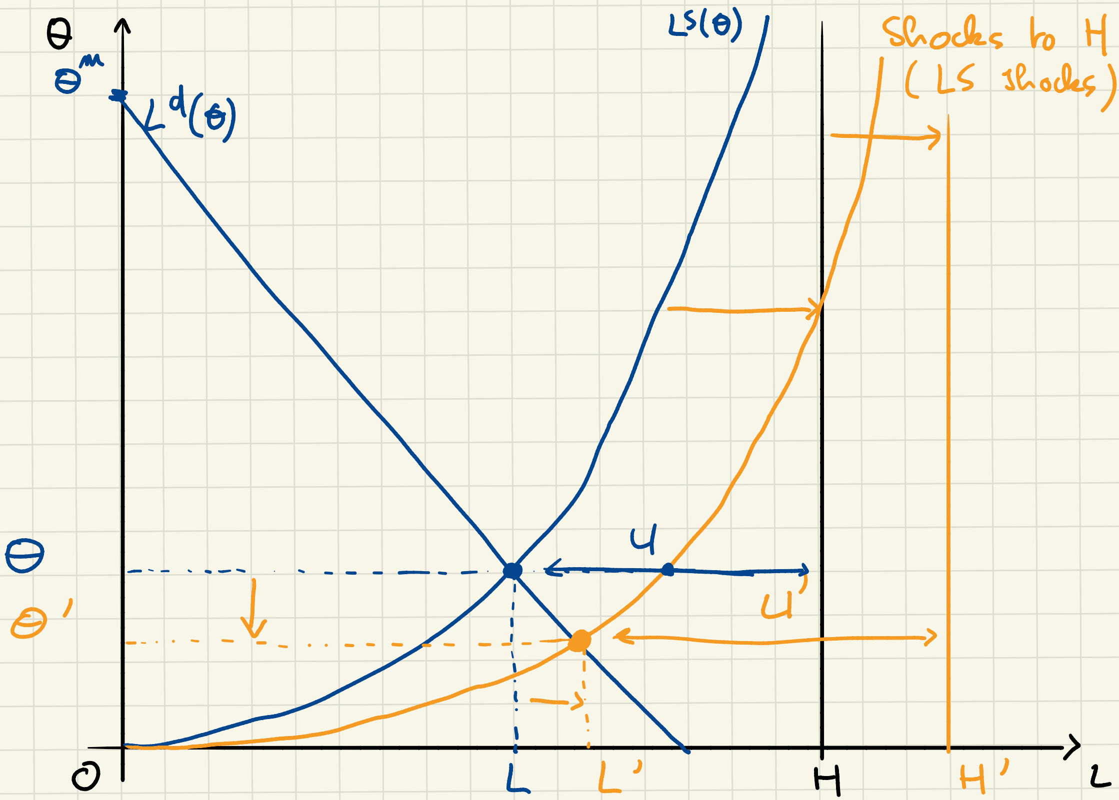
→ gives equilibrium θ.

<u>Which shock can explain fluctuations</u>.

compare <u>labor-demand</u> shock a with <u>labor-supply</u> shock H.

Compare labor-demand & labor-supply shocks from labor-market diagram.

<u>Labor-supply shocks</u>:



increase in H :

- $L \uparrow$: boom / expansion
- $\theta \downarrow$
- $u = s / s + f(\theta)$ so $u \uparrow$
- $Y = a \cdot N^{\alpha}$

$N = L / [1 + \tau(\theta)]$

$\theta \downarrow \Rightarrow \begin{cases} \tau(\theta) \downarrow \\ L \uparrow \end{cases} \Rightarrow N \uparrow \Rightarrow Y \uparrow$

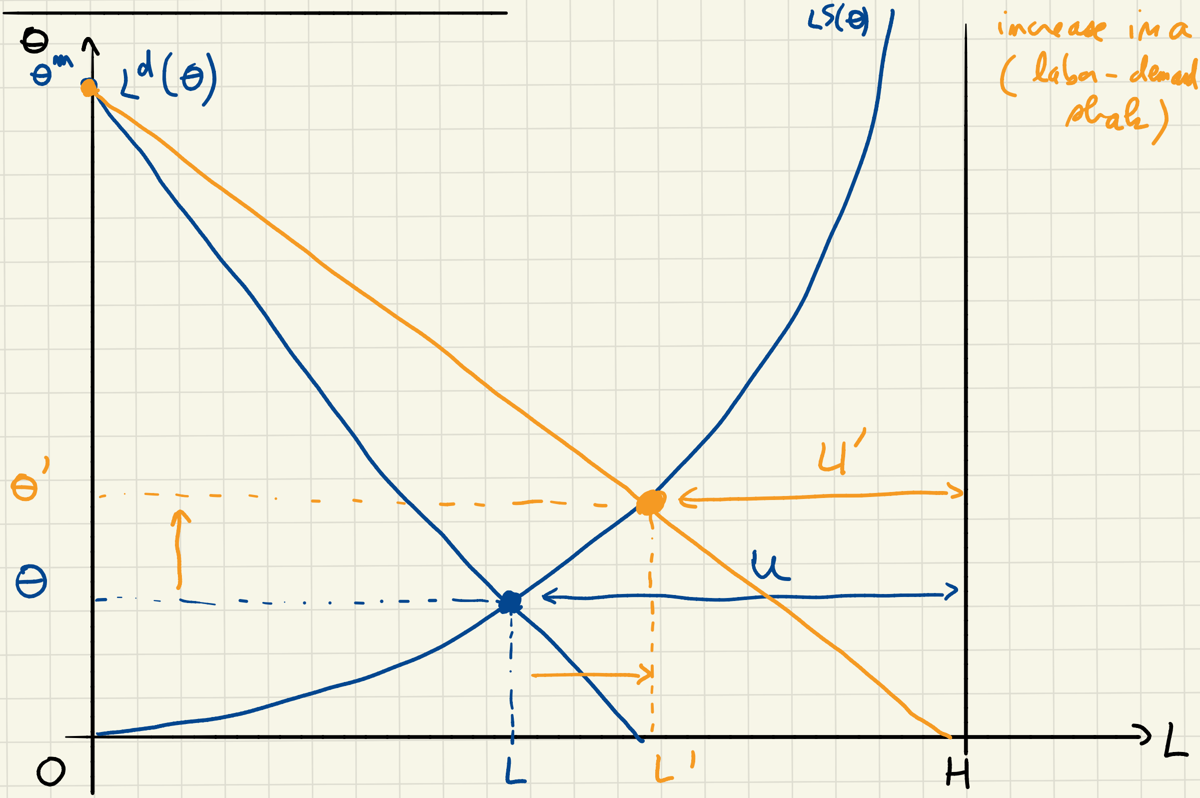
- u is procyclical
- θ is countercyclical

] if H causes business cycles.

$\Rightarrow$ predictions are counterfactual

$\Rightarrow$ H / labor-supply shocks cannot cause business cycles (in matching model)

Labor-demand shocks



increase in a (positive labor-demand shock)

- $L \uparrow$: boom / expansion / peak
- $\theta \uparrow$
- $u = s / s + f(\theta)$ so $u \downarrow$
- $U = u \cdot H$ so $U \downarrow$
- balanced flows : inflows = outflows

$$s \times L = m(U, V)$$
$$\uparrow \qquad\qquad \downarrow \quad \uparrow$$

$V \uparrow$ and $v = V/H \uparrow$

under shocks to labor productivity:

- θ is procyclical
- v is procyclical
- u is countercyclical

Beveridge curve

In matching model: shocks to productivity generate realistic business cycle.
[under rigid wages]

labor-demand shocks.

Michaillat (2012, p. 1741): elasticity of θ wrt to productivity is $\boxed{\varepsilon_a^\theta \approx 8}$.

$\rightarrow$ when productivity goes up by 1%, tightness goes up by 8%.

- elasticity of u wrt a is $\varepsilon_a^u \approx 4$
- elasticity of v wrt a is $\varepsilon_a^v \approx 4$

Can we get ε_a^θ of 8 in matching model with rigid wages?

equilibrium condition: $L^s(\theta) = L^d(\theta)$

$$\Leftrightarrow \frac{f(\theta)}{s+f(\theta)} \cdot H = \left[\frac{\alpha \cdot a^{1-\gamma}}{w\,[1+\tau(\theta)]^\alpha}\right]^{1/1-\alpha}$$

$\rightarrow \theta$ is an implicit function of a: $\theta(a)$

$\rightarrow$ (implicit function theorem)

$$\varepsilon_a^\theta \equiv \frac{d\ln\theta}{d\ln a}.$$

Cobb-Douglas

$$m(U,V) = \mu \cdot U^\eta \cdot V^{1-\eta} \qquad \eta \in (0,1)$$

$$f(\theta) = \mu \cdot \theta^{1-\eta} \rightarrow \frac{d\ln f}{d\ln\theta} = 1-\eta$$

$$q(\theta) = \mu \cdot \theta^{-\eta} \rightarrow \frac{d\ln q}{d\ln\theta} = -\eta$$

$$\frac{f(\theta)}{s+f(\theta)} = 1-u = \ell(\theta) : \underline{\text{employment rate}}.$$

$$\frac{d\ln \ell}{d\ln\theta} = \frac{d\ln f}{d\ln\theta} - \frac{d\ln(s+f)}{d\ln\theta}$$

$$= 1-\eta - \frac{f}{s+f} \times \frac{d\ln f}{d\ln\theta}$$

$$= (1-\eta)\left(1-\frac{f}{s+f}\right)$$

$$= (1-\eta)\times\frac{s}{s+f}$$

$$\boxed{\frac{d\ln \ell}{d\ln\theta} = (1-\eta)\times u}$$

Results on elasticities

Leibniz's notation : $\frac{dx}{df}$

$f(x)$: any differentiable function

$$\boxed{df = f'(x)\,.\,dx} \longrightarrow \frac{df}{dx} = f'(x)$$

elasticity of f with respect to x :

$$\boxed{\varepsilon^f_x = \frac{d\ln f}{d\ln x}}$$

Interpretation

x increases by 1% : what happens to f ?

$$dx = 1\% \times x$$

$$df = f'(x)\,dx = f'(x)\times x \times 1\%$$

percentage change in f :

$$\rightarrow \frac{df}{f} \times 100 = f'(x) \times \frac{x}{f} \times \cancel{1\%} \times \cancel{100}$$

$$\text{percentage change in } f = \frac{x}{f} f'(x) = \frac{x}{f} \cdot \frac{df}{dx}$$

$$d\ln f = \frac{1}{f} \cdot df \quad ; \quad d\ln x = \frac{1}{x} \cdot dx$$

← derivative of ln

$$\boxed{\frac{d\ln f}{d\ln x} = \frac{x}{f} \cdot \frac{df}{dx}} \qquad \boxed{\frac{d\ln f}{d\ln x} = \frac{x}{f} \cdot f'(x)}$$

|| percentage change in f given by $\frac{d\ln f}{d\ln x} = \varepsilon^{f}_{x}$

Results about elasticities :

- $f(x) = x^{\alpha} \longrightarrow \boxed{\varepsilon^{f}_{x} = \alpha}$

proof : $\ln(f) = \alpha \cdot \ln(x) \rightarrow \frac{d\ln f}{d\ln x} = \alpha$

- $f(x) = A(x) \times B(x) \longrightarrow \boxed{\varepsilon^{f}_{x} = \varepsilon^{A}_{x} + \varepsilon^{B}_{x}}$

proof : $\ln f = \ln A + \ln B$

$$\frac{d\ln f}{d\ln x} = \frac{d\ln A}{d\ln x} + \frac{d\ln B}{d\ln x}$$

• $f(x) = A(x) / B(x) \longrightarrow \boxed{\varepsilon^f_x = \varepsilon^A_x - \varepsilon^B_x}$

proof: $\ln f = \ln A - \ln B$

$$\frac{d\ln f}{d\ln x} = \frac{d\ln A}{d\ln x} - \frac{d\ln B}{d\ln x}$$

• $f(x) = \alpha \cdot A(x) \longrightarrow \boxed{\varepsilon^f_x = \varepsilon^A_x}$

proof: $\ln f = \ln \alpha + \ln A$

$$\underbrace{\frac{d\ln f}{d\ln x}}_{\varepsilon^f_x} = \underbrace{\frac{d\ln \alpha}{d\ln x}}_{0} + \underbrace{\frac{d\ln A}{d\ln x}}_{\varepsilon^A_x}$$

• $f(x) = A(x) + B(x) \longrightarrow \boxed{\varepsilon^f_x = \frac{A}{A+B} \cdot \varepsilon^A_x + \frac{B}{A+B} \cdot \varepsilon^B_x}$

"elasticity of sum = weighted sum of elasticities"

proof: $\ln f = \ln (A+B)$

$$\frac{d\ln f}{d\ln x} = \frac{1}{A+B} \times \left(\frac{dA}{d\ln x} + \frac{dB}{d\ln x} \right)$$

$$= \frac{A}{A+B} \cdot \frac{1}{A} \cdot \frac{dA}{d\ln x} + \frac{B}{A+B} \cdot \frac{1}{B} \cdot \frac{dB}{d\ln x}$$

$d\ln A = \frac{dA}{A}$; $d\ln B = \frac{dB}{B}$

$$\frac{d\ln f}{d\ln x} = \underbrace{\frac{A}{A+B}}\cdot \underbrace{\frac{d\ln A}{d\ln x}}_{\varepsilon^A} + \frac{B}{A+B}\,\underbrace{\frac{d\ln B}{d\ln x}}_{\varepsilon^B}$$

$\uparrow \varepsilon f$

- $\underbrace{1+\tau(\theta)}_{g(\theta)}$: $\varepsilon^g_\theta = \frac{\tau}{1+\tau}\cdot \varepsilon^\tau_\theta$

- $\tau(\theta) = \frac{rs}{q(\theta)-rs}$ $\qquad \varepsilon^\tau_\theta = -\left(\frac{q}{q-rs}\times \varepsilon^q_\theta\right)$

- $\varepsilon^q_\theta = -\eta$

- $\varepsilon^\tau_\theta = \eta \cdot \frac{q}{q-rs} = \eta\,(1+\tau)$

- $\varepsilon^g_\theta = \frac{\tau}{1+\tau}\cdot \eta \cdot (1+\tau)$

$$\Rightarrow \boxed{\varepsilon^{1+\tau}_\theta = \tau\cdot\eta}$$

equilibrium: $\frac{f(\theta)}{1+f(\theta)}\cdot H = \left[\frac{\alpha\cdot a^{1-\gamma}}{\omega\,[1+\tau(\theta)]^\alpha}\right]^{\frac{1}{1-\alpha}}$

$\theta(a)$. - Consider small change in productivity $d\ln a$

$\rightarrow \quad d\ln\theta$: induced, small change in θ

$$\boxed{df = f'(x)\,dx} \rightarrow \frac{df}{f} = \frac{1}{f} f'(x)\, dx$$

$$\rightarrow \frac{df}{f} = \frac{x}{f} \cdot f'(x)\, \frac{dx}{x}$$

$$\rightarrow \boxed{d\ln f = \varepsilon_x^f \cdot d\ln x}$$

extends to multivariate functions.

- $$d\ln L^s = \varepsilon_\theta^{L^s} \cdot d\ln\theta$$

$$L^s = \frac{f(\theta)}{s + f(\theta)} \cdot H \rightarrow \varepsilon_\theta^{L^s} = (1-\eta) \cdot u$$

$$\boxed{d\ln L^s = (1-\eta) \cdot u \cdot d\ln\theta}$$

- $$d\ln L^d(\theta, a) = \varepsilon_\theta^{L^d} \cdot d\ln\theta + \varepsilon_a^{L^d}\, d\ln a$$

$$\left(df(x,y) = \frac{\partial f}{\partial x}\, dx + \frac{\partial f}{\partial y}\, dy \right)$$

$$L^d(\theta, a) = \left[\frac{\alpha \cdot a^{1-\gamma}}{\omega \cdot (1 + \tau(\theta))^{\alpha}} \right]^{1/1-\alpha}$$

$$\rightarrow \varepsilon_a^{L^d} = \frac{1}{1-\alpha} \cdot 1-\gamma = \frac{1-\gamma}{1-\alpha}$$

$$\rightarrow \varepsilon_\theta^{L^d} = \frac{-1}{1-\alpha} \cdot \alpha \cdot \varepsilon_\theta^{1+\tau}$$

$$\varepsilon_{\theta}^{L^d} = \frac{-\alpha}{1-\alpha} \cdot \eta \cdot \tau$$

$$\boxed{d\ln L^d = \frac{1-\gamma}{1-\alpha} \, d\ln a - \frac{\alpha}{1-\alpha} \cdot \eta \cdot \tau \cdot d\ln \theta}$$

before shock: $L^s(\theta) = L^d(\theta, a)$

after shock: $L^s(\theta') = L^d(\theta', a')$

$$d\ln L^s = d\ln L^d$$

$$(1-\eta) u \; d\ln \theta = \frac{1-\gamma}{1-\alpha} \, d\ln a - \frac{\alpha}{1-\alpha} \eta \tau \, d\ln \theta$$

$$(1-\eta) u \frac{d\ln \theta}{d\ln a} = \frac{1-\gamma}{1-\alpha} - \alpha/_{1-\alpha} \cdot \eta \cdot \tau \frac{d\ln \theta}{d\ln a}$$

$$\left[(1-\eta) u + \frac{\alpha}{1-\alpha} \eta \tau\right] \frac{d\ln \theta}{d\ln a} = \frac{1-\gamma}{1-\alpha}$$

$$\boxed{\frac{d\ln \theta}{d\ln a} = \frac{1-\gamma}{(1-\alpha)(1-\eta) u + \alpha \eta \tau}}$$

- $\gamma = 1$ (wage $\propto$ a : flexible wage)

  $\frac{d\ln \theta}{d\ln a} = 0$: no fluctuations

  $\rightarrow$ need somewhat rigid wage to obtain business-cycle fluctuations $\rightarrow$ $\gamma < 1$.

- $\gamma < 1$ (wages are rigid) : $\frac{d\ln \theta}{d\ln a} > 0$

→ necessity of wage rigidity to obtain business cycle fluctuations in matching model

→ in data : wages are rigid ($\gamma \simeq 0.5$) so model can generate business-cycle fluctuations.

Calibration of model :

γ :	0.5
η :	0.5
α :	2/3
u :	6 %
τ :	3 %

$$\varepsilon_a^\theta = \frac{1-\gamma}{(1-\alpha)(1-\eta)\,u + \alpha\,\eta\,\tau}$$

$$= \frac{\cancel{0.5}}{1/3 \cdot \cancel{0.5} \cdot 6\% + 2/3 \cdot \cancel{0.5} \cdot 3\%}$$

$$= \frac{1}{2\% + 2\%}$$

$$\varepsilon_a^\theta = \frac{1}{4\%}$$

$$\boxed{\varepsilon_a^\theta = 25}$$

if we want $\varepsilon_a^\theta = 8$: what is γ ?

$$8 = \frac{1-\gamma}{1/3 \cdot 1/2 \cdot 6\% + 2/3 \cdot 1/2 \cdot 3\%}$$

$$8 \times [\,1\% + 1\%\,] = 1 - \gamma$$

$$16\% = 1 - \gamma$$

$$\boxed{\gamma = 0.84}$$

Matching model with bargained wage

Wage bargaining solution: surplus sharing

$$W = (1-\beta)\, z + \beta \cdot MPL \cdot (1 + r\theta)$$

$$MPL = \alpha \cdot a \cdot N^{\alpha-1}$$

Linear production function: $Y = a \cdot N$

$\alpha = 1$

$$\rightarrow \quad MPL = a$$

$$\rightarrow \boxed{W = (1-\beta)\, z + \beta a (1 + r\theta)}$$

Labor demand: $$\left(L^{d}\right)^{1-\alpha} = \left[\frac{\alpha \quad a}{w\,[1+\tau(\theta)]^{\alpha}}\right]^{\cancel{\frac{1-\alpha}{1-\alpha}}}$$

As we set $\alpha = 1$: $$1 = \frac{a}{w \cdot [1+\tau(\theta)]}$$

Labor demand relation:

$$\boxed{\underbrace{a}_{MPL} = \underbrace{[1+\tau(\theta)] \cdot W}_{MC}}$$

Combined bargained wage & labor demand:

$$a = [1+\tau(\theta)] \times \left[(1-\beta)\, z + \beta a (1 + r\theta)\right]$$

$$\boxed{1 = [1+\tau(\theta)]\left[(1-\beta)\frac{z}{a} + \beta(1+r\theta)\right]}$$

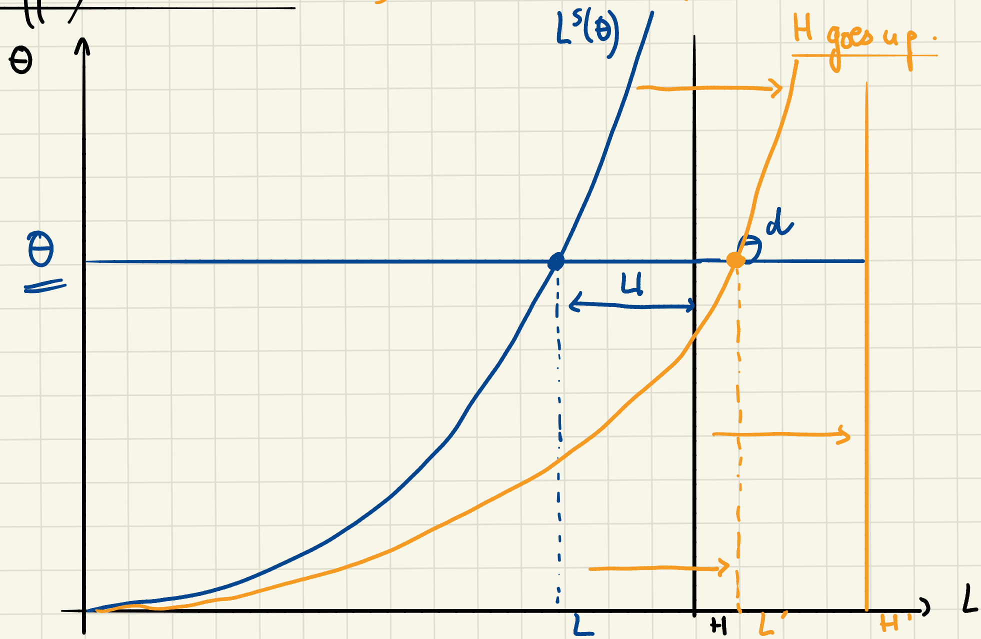
↳ new labor demand: linear prod. function + bargained wage.

↳ • perfectly elastic labor demand

• horizontal labor demand.

↳ labor demand $\theta^d(a)$

<u>Labor supply shock:</u> shock to H



<u>H goes up:</u>

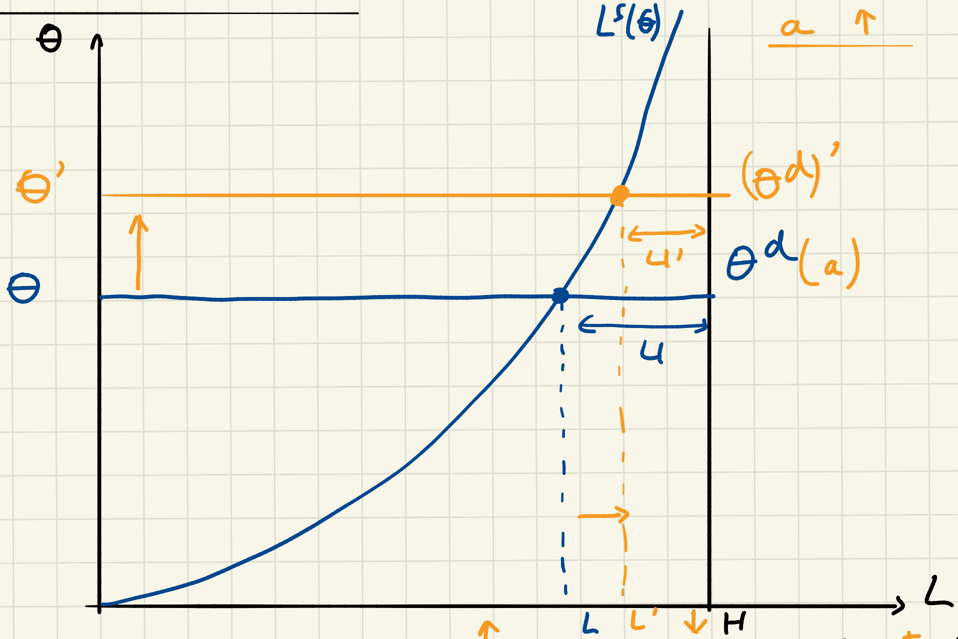
- $L \uparrow$: boom / expansion ($Y \uparrow$)
- $\theta \rightarrow$
- $u = s/[s+f(\theta)]$ rate u remains same
- $U = u \times H$ $U \uparrow$
- $v = \theta \times u$ v remains same
- $V = v \times H$ $V \uparrow$

$\rightarrow$ θ, u, v are acyclical

$\rightarrow$ not a realistic business cycle

Labor demand shock : shock to a

$a \uparrow$



Labor demand θ^d : $1 = [1+\tau(\theta)]\left[(1-\beta)\frac{z}{a} + \beta(1+r\theta)\right]$

if $a \uparrow$: $\theta^d(a) \uparrow$

After an increase in a:

- $L \uparrow$: boom / expansion
- $\theta \uparrow$: θ procyclical
- $\begin{cases} u \downarrow \\ U \downarrow \end{cases}$: u countercyclical
- $\underset{\text{(inflows)}}{s \times \overset{\uparrow}{L}} = m(\underset{\downarrow}{U}, \underset{\text{(outflows)}}{\overset{}{V \uparrow}})$

$\rightarrow \begin{cases} V \uparrow \\ v \uparrow \end{cases}$: v procyclical

Under labor-demand shocks, fluctuations have realistic features (qualitatively)

Not the case with labor-supply shocks.

Objective: compute ε_a^{θ}.

Labor demand relation:

$$\underset{RHS(\theta,a)}{1} = [1 + \tau(\theta)]\left[(1-\beta)\frac{z}{a} + \beta(1 + r\theta)\right]$$

Study impact of an infinitesimal change in productivity: $d\ln a$

$\rightarrow$ Generates infinitesimal change in tightness:
$d\ln \Theta$

$$d\ln RHS = \varepsilon_a^{RHS} \cdot d\ln a + \varepsilon_\theta^{RHS} \cdot d\ln \theta = 0$$

so $$\frac{d\ln\theta}{d\ln a} = -\frac{\varepsilon_a^{RHS}}{\varepsilon_\theta^{RHS}}$$

- First, compute $\varepsilon_a^{RHS} = \dfrac{\partial \ln RHS}{\partial \ln a}$.

$$RHS = [1+\tau(\theta)]\cdot\left[(1-\beta)\frac{z}{a} + \beta(1+r\theta)\right]$$

$$\frac{\partial \ln RHS}{\delta \ln a} = \frac{(1-\beta)(z/a)}{(1-\beta)(z/a)+\beta(1+r\theta)}\cdot(-1)$$

$$\frac{\partial \ln RHS}{\partial \ln a} = \frac{-(1-\beta)\,z}{(1-\beta)\,z+\beta a\,(1+r\theta)} = \varepsilon_a^{RHS}.$$

- Second, compute $\varepsilon_\theta^{RHS} = \dfrac{\partial \ln RHS}{\partial \ln \theta}$.

$$\varepsilon_\theta^{RHS} = \eta\cdot\tau(\theta) + \frac{\beta(1+r\theta)}{(1-\beta)(z/a)+\beta(1+r\theta)}$$

$\varepsilon_\theta^{1+\tau}$

$$* \quad \frac{r\theta}{1+r\theta}\cdot 1$$

$$\varepsilon_\theta^{RHS} = \eta \cdot \tau(\theta) + \frac{\beta r \theta a}{(1-\beta)z + \beta a(1+r\theta)}.$$

- Third, combine results to compute ε_a^θ:

$$\frac{d\ln\theta}{d\ln a} = \frac{(1-\beta)z}{\cancel{(1-\beta)z + \beta a(1+r\theta)}} \cdot \frac{\cancel{(1-\beta)z + \beta a(1+r\theta)}}{\beta r\theta a + \eta\tau \cdot \frac{a}{1+\tau}}$$

$$a = (1+\tau)\left((1-\beta)z + \beta a(1+r\theta)\right) \qquad \text{(labor demand)}$$

$$\frac{a}{1+\tau} = (1-\beta)z + \beta a(1+r\theta)]$$

$$\boxed{\frac{d\ln\theta}{d\ln a} = \frac{(1-\beta)\cdot z}{a\cdot\left[\beta r\theta + \eta\cdot\frac{\tau}{1+\tau}\right]}}$$

- z : value from unemployment

$z = 0$: $\frac{d\ln\theta}{d\ln a} = 0$: no business-cycle fluctuations

θ, L, u, v : do not respond to productivity a.

Intuition: if $z = 0$: W is proportional to a

→ w is flexible : W absorbs fluctuations in a so θ^d is independent of a

Algebraically: $z = 0 \Rightarrow w = \beta a (1 + r\theta)$

Labor demand relation: $a = [1+\tau(\theta)]\, w$

$$\Rightarrow 1\, \cancel{a} = [1+\tau(\theta)]\, \beta\, \cancel{a}\, (1+r\theta)$$

$$\Rightarrow 1 = [1+\tau(\theta)]\, \beta\, (1+r\theta)$$

$\hookrightarrow$ independent of a

Shimer (2005):

$z = 0.4$

$a = 1$ (normalization)

$\eta = 0.5$

$\beta = 0.5$ (tradition)

$\tau = 3\%$

$r\theta = 0.6$

Labor demand relation:

$$a = (1+\tau)\left((1-\beta) z + \beta a (1+r\theta)\right)$$

$$1 = (1.03)\left(\tfrac{1}{2} \cdot 0.4 + \tfrac{1}{2} \cdot 1 \cdot (1+r\theta)\right)$$

$$1 = (1.03)\left(0.2 + 0.5\,(1+r\theta)\right)$$

$$r\theta = \left[\frac{1}{1.03} - 0.2\right] \times 2 - 1 \simeq 0.6$$

Calibrated value of $\dfrac{d\ln\theta}{d\ln a}$?

$$\frac{d\ln\theta}{d\ln a} = \frac{0.4 \times 0.5}{0.5 \times 0.6 + 0.5 \cdot 0.03/1.03} = \frac{0.2}{0.3 + 0.015}$$

$$\frac{d\ln\theta}{d\ln a} \simeq 2/3 > 0 \quad \text{once } \underline{z > 0}$$

BUT $\boxed{\dfrac{d\ln\theta}{d\ln a} << 8}$

Fluctuations in θ, u, v are much smaller than in US data.

→ Model with surplus sharing as wage function is inappropriate to describe business cycle on labor market.

→ violates criterion # 1 for a good model (Kuhn)

See T. Bewley (1999).