

Definition and Properties of the Household's Problem

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Household maximizes utility, subject to budget constraint, taking as given price of services p & market tightness x .

$$\max_{c, m} u(c, m/p)$$

$$\text{s.t. } m + p[1 + \tau(x)] \cdot c - \mu - p f(x) \cdot k = 0$$

Rewrite budget constraint:

$$\frac{m}{p} = \frac{\mu}{p} + f(x) \cdot k - [1 + \tau(x)] c$$

$$\hookrightarrow \max_c u\left(c, \frac{\mu}{p} + f \cdot k - (1 + \tau) \cdot c\right)$$

$$\max_c \frac{x}{1+x} c^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+x} \left[\frac{\mu}{p} + f \cdot k - (1 + \tau) c \right]^{\frac{\varepsilon-1}{\varepsilon}}$$

$\varepsilon > 1 \rightarrow$ concave inc

- $y \mapsto y^{\frac{\varepsilon-1}{\varepsilon}} / (1+x)$ is strictly concave in y , and increasing

- $c \mapsto \frac{\mu}{p} + f \cdot k - (1 + \tau)c$ is linear \Rightarrow concave inc

- composition of concave & increasing \wedge jointly concave function \rightarrow concave inc

concave in $c \geq 0$