How Can a Statistical Agency Predict Tightness?

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Statistical agency announces a tightness x^s

Households take x^s as given

→ Household i will buy $y^i(x^s) = \sigma(x^s)\left[f(x^s)k_i + \frac{\nu_i}{p}\right]$

→ Household i will visit $v^i(x^s) = \frac{y^i(x^s)}{q(x^s)}$

→ Realized tightness will be:

$$x = \frac{\sum_i v_i(x^s)}{\sum_i k_i} = \frac{\sum_i y_i(x^s)}{q(x^s)\cdot \bar{k}}$$

$$x = \frac{\sigma(x^s)\left(f(x^s)\,k + \nu/p\right)}{q(x^s)\,k}$$

$$\frac{x}{x^s} = \frac{\sigma(x^s)\left[\overbrace{f(x^s)\,k}^{y^s(x^s)} + \nu/p\right]}{\underbrace{x^s\, q(x^s)}_{f(x^s)}\cdot k}$$

$\frac{x}{x^s}$ must be 1

$y^s(x^s)$

Statistical agency aims to make a correct forecast:

They aim to announce x^s such $x^s = x$

They announce x^S such that

$$y^S(x^S) = \sigma(x^S)\left[y^S(x^S) + \nu/p\right]$$

$$\left[1 - \sigma(x^S)\right] y^S(x^S) = \sigma(x^S)\,(\nu/p)$$

$$y^S(x^S) = \underbrace{\frac{\sigma(x^S)}{1-\sigma(x^S)}\,\frac{\nu}{p}}_{y^d(x^S)}$$

Statistical agency announces x^S such that:

$$\boxed{y^S(x^S) = y^d(x^S)}$$

Since x (realized) $= x^S$ (forecast)

then

$$\boxed{y^S(x) = y^d(x)}$$