Solution of the Model in a Special Case Without Matching Cost

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Special case:

- No matching cost: $\rho = 0$, $\tau = 0$
- Matching function: $\gamma = 1$, $f(x) = \frac{x}{1+x}$, $q(x) = \frac{1}{1+x}$

$$m = \left[\frac{1}{v} + \frac{1}{k}\right]^{-1} = \frac{1}{\frac{1}{v} + \frac{1}{k}}$$

Solution of the model: Find (v, x) such that

$$\begin{cases} v = x \cdot k \\ v = \frac{\sigma(x)}{q(x)} \left[f(x) k + \frac{\mu}{\rho} \right] \end{cases}$$

$\sigma(x) = \frac{x^{\varepsilon}}{1+x^{\varepsilon}}$ b/c $\tau(x) = 0$

$$\begin{cases} v = x \quad k \\ v = \sigma \cdot \left[\frac{f(x)}{q(x)} k + \frac{\mu}{\rho \cdot q(x)} \right] \end{cases}$$

x

$\frac{1}{q(x)} = 1 + x$

$$\Leftrightarrow \begin{cases} v = k \quad x \\ v = \sigma \left[k \quad x + \frac{\mu}{\rho} (1+x) \right] \end{cases}$$

$$\Leftrightarrow \begin{cases} v = k \cdot x \\ v = \underbrace{\sigma \cdot \left(k + \frac{\mu}{\rho}\right)}_{\lambda} x + \frac{\sigma\mu}{\rho} \end{cases}$$

$$\Leftrightarrow \begin{cases} v = k \cdot x \\ v = \lambda \cdot x + \frac{\sigma\mu}{\rho} \end{cases}$$

Is λ smaller or larger than k?

To have a solution, amount of services demanded is less than capacity $\rightarrow$ $\boxed{y^d = \frac{\sigma}{1-\sigma}\frac{\mu}{\rho} < k}$

$\Rightarrow \sigma \frac{\mu}{\rho} < (1-\sigma) k$

$\Rightarrow \frac{\mu}{\rho} < \left(\frac{1-\sigma}{\sigma} \cdot k\right)$

$\Rightarrow \lambda < \sigma\left(k + \frac{1-\sigma}{\sigma} k\right)$

$\Rightarrow \lambda < \sigma k + (1-\sigma) k = k$

$\Rightarrow \boxed{\lambda < k}$

Solution of model